J. Fluid Mech. (1971), *vol.* 46, *part* 1, *pp.* 129–139
Printed in Great Britain

Wave forces on a circular dock

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(Received 14 April 1970)

The scattering of surface gravity waves by a circular dock is considered in order to determine the horizontal and vertical forces and torque on the dock. An incident plane wave is expanded in Bessel functions, and for each mode the problem is formulated in terms of the potential on the cylindrical surface containing the dock and extending to the bottom. The solution is shown to have phase independent of depth and so may be obtained from an infinite set of real equations, which are solved numerically by Galerkin's method. The convergence of the solution is discussed, and some numerical results are presented.

This problem has been investigated previously by Miles & Gilbert (1968) by a different method, but their work contained errors.

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1. Introduction

The proposal to build an artificial island offshore from the Scripps Institution of Oceanography has motivated a study of the forces that would be exerted on a circular dock by surface waves. Miles & Gilbert (1968) formulated the problem of the scattering of surface waves by a circular dock and obtained a variational approximation to the far field. They used the result to scale their trial function, from which they then calculated the near field, and hence the peripheral disturbance at the dock and the horizontal force exerted on the dock.

Their numerical calculations of the horizontal force were found by W. H. Munk to be incorrect, as they had the wrong asymptotic behaviour for small wave-number. Algebraic and computational errors aside, however, a more fundamental error is pointed out in the appendix of the present paper. Admittedly this error does not affect a variational approximation to the far field, but it greatly affects the calculation of the vertical force. Moreover, it was felt that Miles & Gilbert's method of approximating the near field was somewhat dubious (see the appendix). It has been thought advisable to investigate the problem afresh.

At first sight, it would appear that a full solution for the near field in the scattering problem is necessary for a precise evaluation of the wave forces on a body. However, Haskind (1957) has used Green's theorem to demonstrate how the wave forces in the scattering problem may be calculated from the far field radiated by the body oscillating in otherwise calm water in a mode corresponding to the force required. Newman (1962) discussed this method, and used approximations to the far field in the radiation problem to obtain forces in the scattering problem for submerged ellipsoids and floating elliptical plates. Indeed, the main use of this elegant technique is in problems where the detailed scattering problem is intractable, but where a reasonable approximation for the far field in the radiation problem may be obtained.

However, if one wishes to derive precise values for the forces, the effort required for a full solution of the scattering problem is no greater than for the radiation problem, and of course provides other details, such as the peripheral disturbance at the body, should these be required. The simple geometry of a circular dock does allow one to derive precise solutions without too much trouble, and so in this paper the scattering problem will be formulated, certain general results about the solution will be proved, and some numerical results will be presented.

2. Formulation

The notation used here will be largely the same as that used by Miles & Gilbert (1968). A plane wave of amplitude ζ_0, frequency σ and wave-number k is incident on a circular dock of radius a and draft $d-h$ in water of depth d (see figure 1). σ and k are related by the dispersion relation

$$\sigma^2 = gk \tanh kd. \tag{2.1}$$

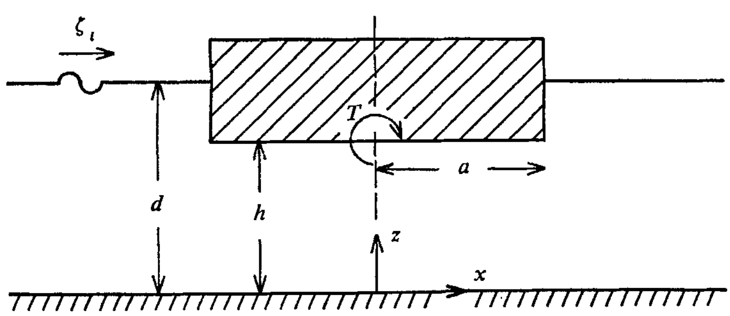

FIGURE 1. Definition sketch. The axis and direction of the torque T (see §4) are shown.

Small amplitude irrotational flow is assumed. The free surface displacement is given by the real part of $\zeta \exp(-i\sigma t)$, where the incident wave has

$$\zeta = \zeta_0 \exp(ikx) \tag{2.2}$$

$$= \zeta_0 \sum_{m=0}^{\infty} \epsilon_m i^m J_m(kr) \cos m\theta, \tag{2.3}$$

in which
$$\epsilon_0 = 1, \quad \epsilon_m = 2 \quad (m \geqslant 1). \tag{2.4}$$

The total disturbance may be expressed as

$$\zeta(r,\theta) = \zeta_0 \sum_{m=0}^{\infty} \epsilon_m i^m \chi_m(r) \cos m\theta, \tag{2.5}$$

and the corresponding displacement potential (the velocity potential $\times\, i/\sigma$) is

$$\phi(r,\theta,z) = \zeta_0 \sum_{m=0}^{\infty} \epsilon_m i^m \psi_m(r,z) \cos m\theta, \tag{2.6}$$

where
$$\chi_m(r) = \left.\frac{\partial \psi_m}{\partial z}\right|_{z=d} \tag{2.7}$$

in order to satisfy the kinematic free-surface condition. r, θ, z are cylindrical co-ordinates; $\theta = 0$ corresponds to the positive x-axis, z is positive upwards with origin on the sea bottom. ϕ must satisfy

$$\nabla^2 \phi = 0, \tag{2.8}$$

$$\sigma^2 \phi - g \frac{\partial \phi}{\partial z} = 0 \quad \text{on} \quad z = d, \tag{2.9}$$

$$\frac{\partial \phi}{\partial z} = 0 \quad \text{on} \quad z = 0, \tag{2.10}$$

$$\frac{\partial \phi}{\partial r} = 0 \quad \text{on} \quad r = a \quad \text{for} \quad h \leqslant z \leqslant d. \tag{2.11}$$

Moreover, ψ_m and $\partial \psi_m / \partial r$ are continuous at $r = a$ for $0 \leqslant z < h$. Miles & Gilbert (1968) expressed ψ_m in the interior region $r \leqslant a$ and the exterior region $r \geqslant a$ in terms of $\partial \psi_m / \partial r|_{r=a}$, and then matched ψ_m at $r = a$. However, it turns out to be rather simpler to express the interior and exterior solutions in terms of $\psi_m|_{r=a}$ and apply conditions on $\partial \psi_m / \partial r$ at $r = a$. Let

$$\frac{1}{d} \psi_m(a, z) = \mathbf{f}_m(z) \quad \text{for} \quad 0 \leqslant z \leqslant d,\dagger \tag{2.12}$$

so that $\mathbf{f}_m(z)$ is dimensionless. For the interior solution we expand $\mathbf{f}_m(z)$ as

$$\mathbf{f}_m(z) = \sum_{n=0}^{\infty} \epsilon_n \mathbf{F}_{mn} \cos\left(\frac{n\pi z}{h}\right) \quad \text{for} \quad 0 \leqslant z < h, \tag{2.13}$$

where
$$\mathbf{F}_{mn} = \frac{1}{h} \int_0^h \mathbf{f}_m(z) \cos\left(\frac{n\pi z}{h}\right) dz. \tag{2.14}$$

Thus

$$\frac{1}{d} \psi_m(r, z) = \mathbf{F}_{m0} \left(\frac{r}{a}\right)^m + 2 \sum_{n=1}^{\infty} \mathbf{F}_{mn} \frac{I_m(n\pi r/h)}{I_m(n\pi a/h)} \cos\left(\frac{n\pi z}{h}\right) \quad \text{in} \quad 0 \leqslant z < h, \quad r \leqslant a, \tag{2.15}$$

where I_m is the modified Bessel function of the first kind. We note that there is no term proportional to $\log r$ in ψ_0 as there is no source or sink under the dock. However, the term $\mathbf{F}_{00}$ is non-zero and vital.

In the exterior region, the appropriate expansion for $\mathbf{f}_m(z)$ is

$$\mathbf{f}_m(z) = \sum_{\alpha} \mathscr{F}_{m\alpha} Z_\alpha(z) \quad \text{in} \quad 0 \leqslant z \leqslant d, \tag{2.16}$$

where
$$\mathscr{F}_{m\alpha} = \frac{1}{d} \int_0^d \mathbf{f}_m(z) Z_\alpha(z) dz. \tag{2.17}$$

α is a root of
$$\alpha \tan \alpha d + \sigma^2/g = 0. \tag{2.18}$$

$\sum_{\alpha}$ denotes summation over α, including $\alpha = -ik$, with corresponding suffix k, as the first term as well as all the positive real roots of (2.18). The eigenfunctions $Z_\alpha(z)$ are given by

$$Z_k(z) = N_k^{-\frac{1}{2}} \cosh kz, \tag{2.19}$$

† Bold face is used to distinguish the potential formulation from the velocity formulation; see Miles (1971).

$$Z_\alpha(z) = N_\alpha^{-\frac{1}{2}} \cos \alpha z, \tag{2.20}$$

where
$$N_k = \tfrac{1}{2}\left[1 + \frac{\sinh 2kd}{2kd}\right], \tag{2.21}$$

$$N_\alpha = \tfrac{1}{2}\left[1 + \frac{\sin 2\alpha d}{2\alpha d}\right]. \tag{2.22}$$

$Z_k(z)$, $Z_\alpha(z)$ then form a complete orthogonal set in $[0, d]$ with mean square values of 1. Hence,

$$\frac{1}{d}\psi_m(r,z) = \left\{J_m(kr) - \frac{J_m(ka)}{H_m(ka)} H_m(kr)\right\} \frac{Z_k(z)}{dZ_k'(d)}$$
$$+ \sum_\alpha \mathscr{F}_{m\alpha} \frac{K_m(\alpha r)}{K_m(\alpha a)} Z_\alpha(z) \quad \text{in} \quad 0 \leqslant z \leqslant d, \quad r \geqslant a. \tag{2.23}$$

J_m is an ordinary Bessel function, and $H_m = J_m + iY_m$ is the Hankel function of the first kind (omitting the usual superscript 1). K_m is the modified Bessel function of the second kind. The first terms on the right-hand side of (2.23) describe the incident wave plus a scattered wave introduced to give a zero net contribution to $\psi_m(a, z)$. Of course the rest of the scattered far field comes from the first term in the infinite sum over α, for which $\alpha = -ik$ and

$$K_m(-ikr) = \tfrac{1}{2}\pi i^{m+1} H_m(kr). \tag{2.24}$$

We now require $\partial\psi_m/\partial r$ to be continuous at $r = a$ for $0 \leqslant z < h$, and to vanish at $r = a$ for $h \leqslant z \leqslant d$ (from 2.11). Hence we obtain the two equations

$$B_m Z_k(z) = \sum_\alpha \mathscr{G}_{m\alpha}^{-1} \mathscr{F}_{m\alpha} Z_\alpha(z) + \sum_{n=0}^{\infty} \epsilon_n G_{mn}^{-1} \mathbf{F}_{mn} \cos\left(\frac{n\pi z}{h}\right) \quad \text{in} \quad 0 \leqslant z < h, \tag{2.25}$$

$$B_m Z_k(z) = \sum_\alpha \mathscr{G}_{m\alpha}^{-1} \mathscr{F}_{m\alpha} Z_\alpha(z) \quad \text{in} \quad h \leqslant z \leqslant d, \tag{2.26}$$

where
$$B_m = -2i[\pi H_m(ka)\, dZ_k'(d)]^{-1}, \tag{2.27}$$

using
$$J_m(ka) H_m'(ka) - J_m'(ka) H_m(ka) = \frac{2i}{\pi ka}. \tag{2.28}$$

Also,
$$\mathscr{G}_{m\alpha} = -K_m(\alpha a)\, [\alpha a K_m'(\alpha a)]^{-1}, \tag{2.29}$$

$$G_{mn} = I_m\left(\frac{n\pi a}{h}\right)\left[\frac{n\pi a}{h} I_m'\left(\frac{n\pi a}{h}\right)\right]^{-1}. \tag{2.30}$$

Now, substituting (2.16) in (2.14), we have

$$\mathbf{F}_{mn} = \sum_\alpha L_{n\alpha} \mathscr{F}_{m\alpha}, \tag{2.31}$$

where
$$L_{n\alpha} = \frac{1}{h}\int_0^h Z_\alpha(z) \cos\left(\frac{n\pi z}{h}\right) dz \tag{2.32}$$

$$= \frac{(-1)^n}{\alpha^2 h^2 - n^2\pi^2} N_\alpha^{-\frac{1}{2}} \alpha h \sin \alpha h, \tag{2.33}$$

and
$$L_{nk} = \frac{(-1)^n}{k^2h^2 + n^2\pi^2} N_k^{-\frac{1}{2}} kh \sinh kh. \tag{2.34}$$

Equation (2.25) now becomes

$$B_m Z_k(z) = \sum_\alpha \mathscr{F}_{m\alpha}\left\{\mathscr{G}_{m\alpha}^{-1} Z_\alpha(z) + \sum_{n=0}^{\infty} \epsilon_n L_{n\alpha} G_{mn}^{-1} \cos\left(\frac{n\pi z}{h}\right)\right\} \quad \text{in} \quad 0 \leqslant z < h. \quad (2.35)$$

An approach to the solution of (2.26) and (2.35) will be described in §3.

3. Solution

If we multiply the two equations (2.26) and (2.35) by $Z_\beta(z)/d$, integrate each over its region of validity and add, we obtain

$$\sum_\alpha E_{\beta\alpha} \mathscr{F}_{m\alpha} = B_m \delta_{k\beta}, \quad (3.1)$$

where

$$E_{\beta\alpha} = \mathscr{G}_{m\beta}^{-1} \delta_{\beta\alpha} + \sum_{n=0}^{\infty} (h/d)\, \epsilon_n L_{n\alpha} L_{n\beta} G_{mn}^{-1}. \quad (3.2)$$

The only imaginary term in $E_{\beta\alpha}$ is contained in

$$\mathscr{G}_{mk}^{-1} = -ka \frac{H'_m(ka)}{H_m(ka)}.$$

Define a real matrix,

$$E_{\beta\alpha}^{(r)} = E_{\beta\alpha} - \mathscr{G}_{mk}^{-1} \delta_{k\alpha} \delta_{k\beta}, \quad (3.3)$$

assume that it is non-singular, and that $\phi_{m\alpha}$ is the solution of

$$\sum_\alpha E_{\beta\alpha}^{(r)} \phi_{m\alpha} = \delta_{k\beta}. \quad (3.4)$$

The solution of (3.1) is then

$$\mathscr{F}_{m\alpha} = \frac{B_m \phi_{m\alpha}}{1 + \mathscr{G}_{mk}^{-1} \phi_{mk}}. \quad (3.5)$$

Of course, $\phi_{m\alpha}$ is real, and so (3.5) gives us the result that the phase of $\mathscr{F}_{m\alpha}$ is independent of α, i.e. the phase of $\mathbf{f}_m(z)$ is independent of z. Garrett (1970) proved a similar result for the problem in which the dock is replaced by an open-bottomed circular cylinder. The numerical solution for $\phi_{m\alpha}$ will be discussed in §5.

4. Wave forces

The various forces on the dock will be calculated from the pressure, given by Bernoulli's theorem as the real part of $p \exp(-i\sigma t)$, where

$$p(r, \theta, z) = \rho\sigma^2 \phi(r, \theta, z) \quad (4.1)$$

$$= \rho\sigma^2 \zeta_0 \sum_{m=0}^{\infty} \epsilon_m i^m \psi_m(r, z) \cos m\theta. \quad (4.2)$$

The force on the dock in the positive x direction is then given by the real part of $X \exp(-i\sigma t)$ where

$$X = -\rho\sigma^2 \int_0^{2\pi} \int_h^d \phi(a, \theta, z) a \cos\theta \, d\theta \, dz \quad (4.3)$$

$$= -2\pi i \rho\sigma^2 \zeta_0 a \int_h^d \psi_1(a, z)\, dz. \quad (4.4)$$

From (2.1), (2.23),

$$X/B = -2ikd\tanh kd[N_k^{-\frac{1}{2}}\mathscr{F}_{1k}(ka)^{-1}(\sinh kd - \sinh kh) + \sum_{\alpha}{}' N_\alpha^{-\frac{1}{2}}\mathscr{F}_{1\alpha}(\alpha a)^{-1}(\sin \alpha d - \sin \alpha h)], \quad (4.5)$$

where

$$B = \pi a^2 \zeta_0 \rho g \quad (4.6)$$

is the hydrostatic buoyancy force associated with a free surface elevation ζ_0 over the area of the dock. $\sum_{\alpha}{}'$ denotes summation over all the real values of α; i.e. $\alpha = -ik$ is omitted from the sum.

The vertical force on the dock is given by the real part of $Z\exp(-i\sigma t)$ where

$$Z = \rho\sigma^2 \int_0^{2\pi}\int_0^a \phi(r,\theta,h) r\,d\theta\,dr \quad (4.7)$$

$$= 2\pi\rho\sigma^2\zeta_0 \int_0^a \psi_0(r,h) r\,dr, \quad (4.8)$$

whence

$$\frac{Z}{B} = 2kd\tanh kd\left[\tfrac{1}{2}\mathbf{F}_{00} + 2\sum_{n=1}^{\infty}(-1)^n\left(\frac{n\pi a}{h}\right)^{-2}\mathbf{F}_{0n}G_{0n}^{-1}\right]. \quad (4.9)$$

The $\mathbf{F}_{0n}$ are obtained from the solution $\mathscr{F}_{0\alpha}$ by (2.31).

The torque on the dock about the axis shown in figure 1 is given by the real part of $T\exp(-i\sigma t)$, where T is made up of T_s, arising from forces on the side of the dock, and T_b, arising from forces on the bottom of the dock.

$$T_s = -2\pi i\rho\sigma^2\zeta_0 a\int_h^d (z-h)\psi_1(a,z)\,dz, \quad (4.10)$$

whence

$$T_s/Ba = -2ikd\tanh kd\{N_k^{-\frac{1}{2}}\mathscr{F}_{1k}(ka)^{-2}[k(d-h)\sinh kd - \cosh kd + \cosh kh] + \sum_{\alpha}{}' N_\alpha^{-\frac{1}{2}}\mathscr{F}_{1\alpha}(\alpha a)^{-2}[\alpha(d-h)\sin\alpha d + \cos\alpha d - \cos\alpha h]\}. \quad (4.11)$$

$$T_b = -2\pi i\rho\sigma^2\zeta_0\int_0^a r^2\psi_1(r,h)\,dr, \quad (4.12)$$

whence

$$\frac{T_b}{Ba} = -2ikd\tanh kd\left[\tfrac{1}{4}\mathbf{F}_{10} + 2\sum_{n=1}^{\infty}(-1)^n\left(\frac{n\pi a}{h}\right)^{-2}\mathbf{F}_{1n}(G_{1n}^{-1}-1)\right], \quad (4.13)$$

in which the $\mathbf{F}_{1n}$ are obtained from the solution $\mathscr{F}_{1\alpha}$ by (2.31).

One interesting result, immediately apparent from these formulae and (3.5), is that the horizontal force and the torque are in phase (apart from a possible difference in sign).

When the dock extends all the way to the bottom (i.e. $h = 0$), the horizontal force and torque are given by the results of MacCamy & Fuchs (1954):

$$\frac{X}{B} = \frac{4}{\pi(ka)^2 H_1'(ka)}\tanh kd, \quad (4.14)$$

$$\frac{T}{Ba} = \frac{T_s}{Ba} = \frac{4}{\pi(ka)^3 H_1'(ka)}[kd\tanh kd - 1 + \operatorname{sech} kd]. \quad (4.15)$$

5. Numerical methods and results

Having proved the constancy of phase of $\mathscr{F}_{m\alpha}$, we could substitute (3.5) back into (2.26) and (2.35) to obtain real equations involving the unknown coefficients $\phi_{m\alpha}$. These equations could then be solved for a finite number of α by minimizing the error in least squares. However, this is algebraically complicated, so it was decided to solve (3.4) taking only the first P values of α and $\phi_{m\alpha}$. This is essentially Galerkin's method. Of course, the infinite series in (3.2) must also be truncated after, say, N terms. For large n the terms in the series fall off like n^{-3}, so that convergence is quite rapid, but the largest contributions for a given α, β come from the values of n close to $\alpha h/\pi$, $\beta h/\pi$. Now the pth real root of (2.18) is close to $p\pi/d$, so that N must be greater than Ph/d in order to include the important terms in the series.

If the forces on the dock obtained from the solution of (3.4) with finite N, P are regarded as functions of N, P, then we require the limit as $N \to \infty$ and $P \to \infty$. Probably the simplest way to check convergence of the results is to solve with a fixed N (or P) for a number of values of P (or N), and extrapolate to infinity by plotting the results against N^{-s} (or P^{-s}) for $s > 0$ and extrapolating to zero. Usually one takes $s = 1$, but if more precision is required, one may generally find two values of s, for one of which the curve is concave, and for the other of which it is convex, and hence upper and lower bounds on the solution are obtained with reasonable confidence. Generally, of course, this extrapolation is only performed for one or two cases, and one then takes N and P large enough so that the forces are given to sufficient accuracy by the results for these values. The time taken for a numerical solution of (3.4) has terms proportional to N and to P^3, and the storage required has terms proportional to N, PN and P^2, so that any need for extrapolation arises with P rather than N. For the values of d/a, h/a and ka considered here I used $P = 20$ and $N = 50$. For the most part this produced extravagantly accurate results, though convergence was slower for the vertical force and the torque due to the forces on the bottom of the dock than for the horizontal force and torque due to the forces on the side of the dock. Errors greater than 1 % only occurred when the vertical force was very small anyway, and it was then easily corrected by extrapolation as a function of $1/P$, taking $P = 10, 20, 30, 40$. The results presented here have all been calculated to an accuracy of 1 % or better.

Figures 2, 3, 4 show the horizontal force, vertical force and torque exerted on the dock as functions of ka for different values of h/a. The (a) figures are for $d/a = 0{\cdot}75$, the (b) figures for $d/a = 1{\cdot}5$. The phase of the horizontal and vertical forces is also shown; the phase of the torque is the same as that of the horizontal force. The phases are plotted as continuous functions (rather than being restricted to $(-\pi, \pi)$), a negative phase represents the amount by which the force leads the incident wave at $x = 0$.

As expected, the horizontal force is in phase with minus the slope of the incident wave for small ka. The maximum horizontal force is roughly proportional to the draft of the dock, $d - h$. As $ka \to \infty$, the force tends to that exerted on a long cylinder in deep water. The phase tends to $-ka + \frac{1}{4}\pi$, i.e. the force lags the wave at the leading edge of the dock by $\frac{1}{4}\pi$.

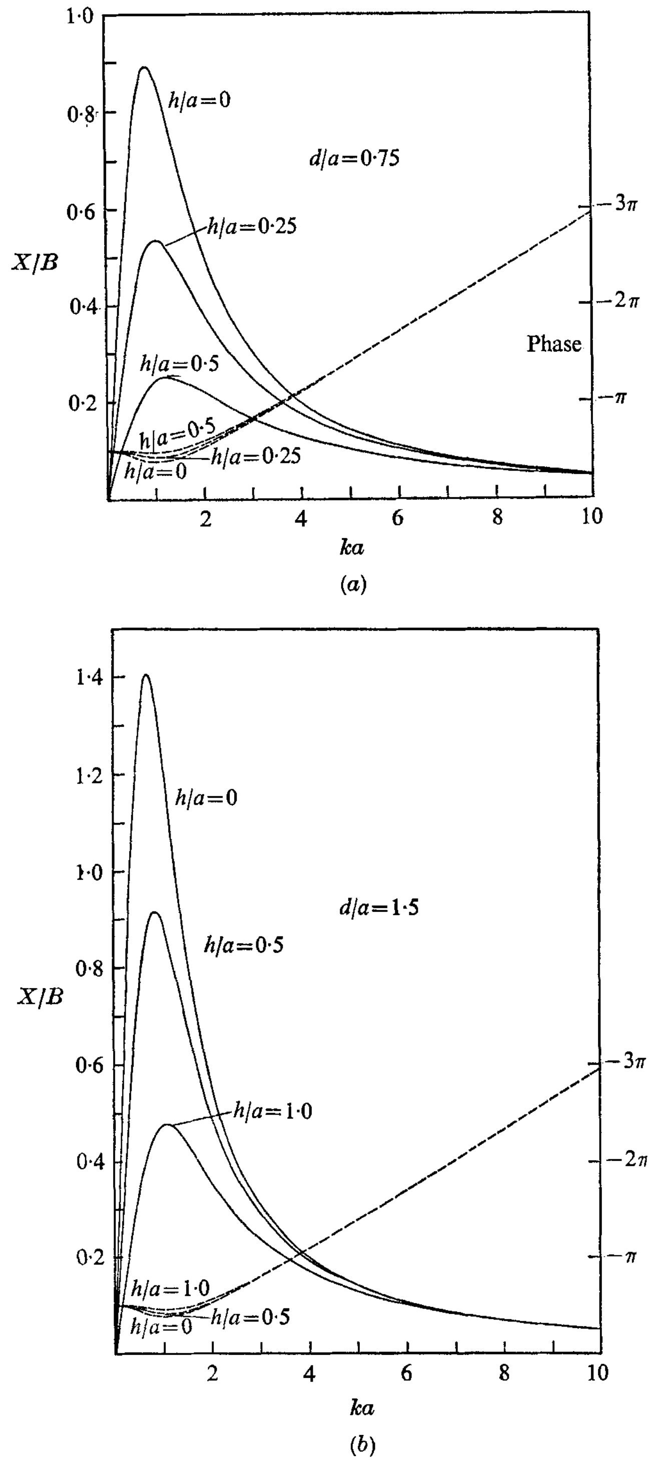

FIGURE 2. Horizontal force on the dock. ——, amplitude; - - -, phase. (a) $d/a = 0{\cdot}75$, (b) $d/a = 1{\cdot}5$.

The vertical force also tends to the right limit for $ka \to 0$; the amplitude tends to the hydrostatic buoyancy force and the phase to zero. As $ka \to \infty$, the phase tends to $-ka + \frac{1}{4}\pi$ as the amplitude tends to zero. The reason for this is that, as the scattering tends to that from a long cylinder, $|\mathscr{G}_{0k}^{-1}\phi_{0k}| \gg 1$ in (3.5), so that the phase of the vertical force equals that of $B_0 G_{0k}$.

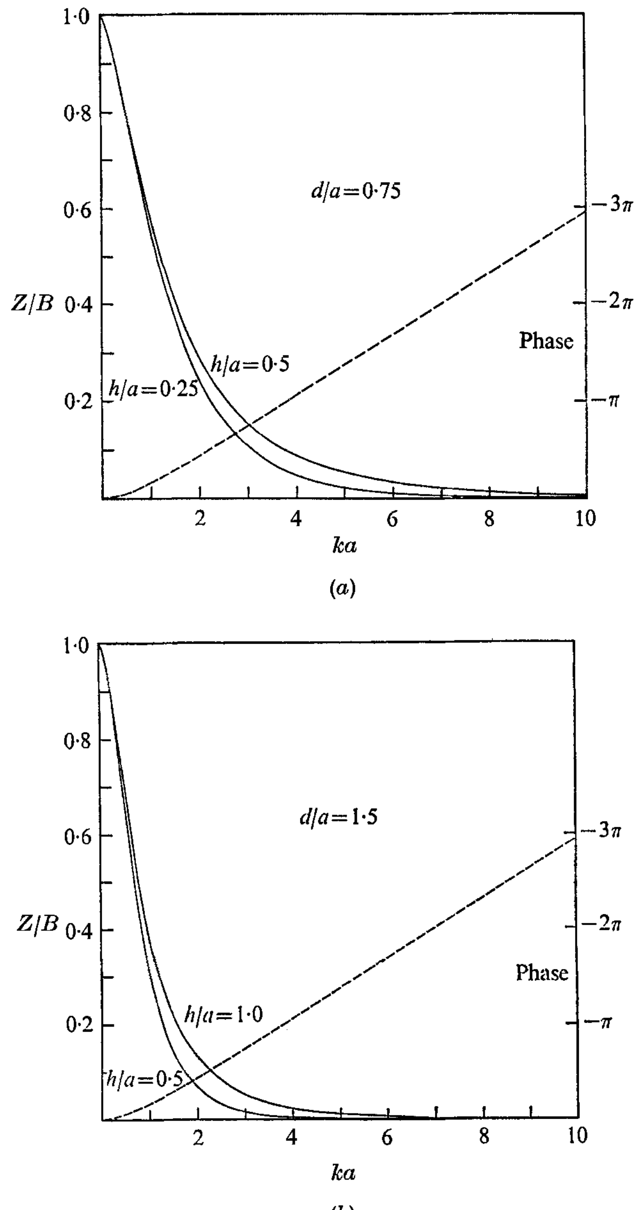

FIGURE 3. Vertical force on the dock. ——, amplitude; - - -, phase. (*a*) $d/a = 0{\cdot}75$, (*b*) $d/a = 1{\cdot}5$. The differences in phase for the two values of h/a are indistinguishable on this scale.

The torque on the dock behaves in much the same way as the horizontal force, being zero at both limits, and having a maximum at a value of ka close to 1. The phase of the torque is always the same as that of the horizontal force, so that for sufficiently large ka all the forces on the dock are in phase. For large ka the torque contributed by the force on the side of the dock behaves like (4.15) with d replaced by $d-h$.

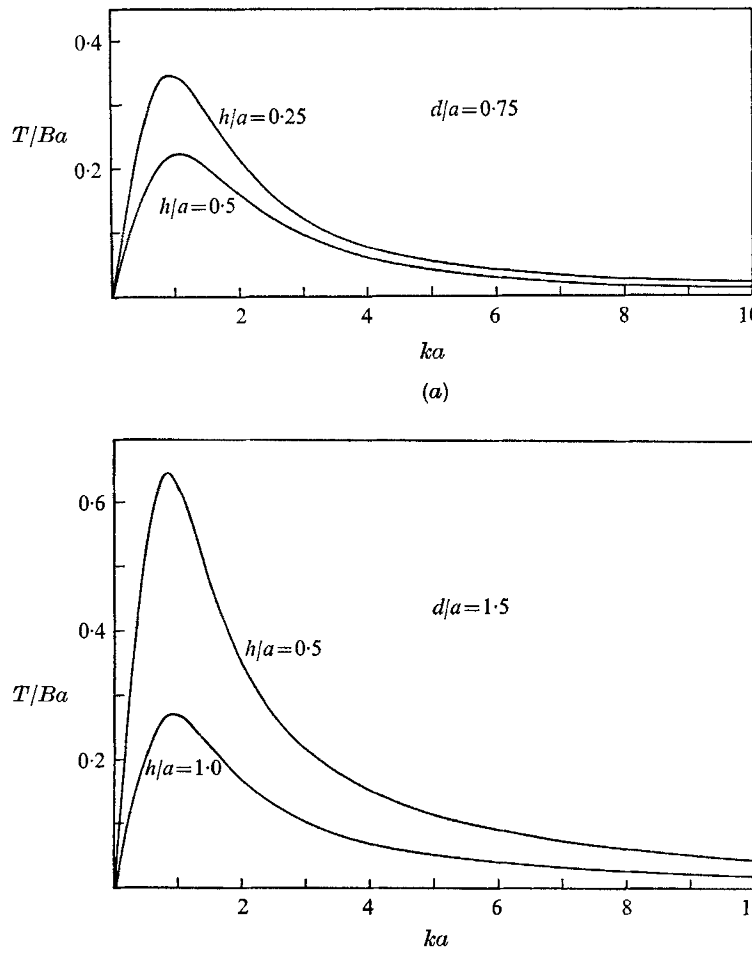

FIGURE 4. Amplitude of torque on the dock. Phase as for the horizontal force. (a) $d/a = 0{\cdot}75$, (b) $d/a = 1{\cdot}5$.

It is interesting to compare the results for $d/a = 0{\cdot}75$, $h/a = 0{\cdot}25$ with those for $d/a = 1{\cdot}5$, $h/a = 1{\cdot}0$, as both describe a dock of draft $(d-h)/a = 0{\cdot}5$. The difference in the horizontal force is small, but the torque is somewhat greater and the vertical force much greater for the smaller value of d/a. This is perhaps understandable in terms of the incident wave trying to squeeze water into a narrower gap between the bottom of the dock and the bottom of the sea for the smaller value of d/a, with greater pressures resulting.

I thank Professors John W. Miles and Freeman Gilbert for discussions of this problem. This work was supported by the National Science Foundation.

Appendix. The formulation of Miles & Gilbert

Miles & Gilbert (1968) investigated the same problem as the present paper. They expanded the interior and exterior solutions in terms of $\partial\psi_m/\partial r|_{r=a}$ and then matched ψ_m at $r = a$ for $0 \leqslant z < h$. As they remark, a constant may be added to their representation of ψ_0 in the interior region (their equation (2.7)). They incorrectly take this constant to be zero. In fact, the constant is related to that part of the pressure under the dock which is independent of position and contributes nearly all the vertical force on the dock. Their (2.20) should thus be

$$F_m Z_k(z) = \int_0^h g_m(z,\zeta) f_m(\zeta)\, d\zeta + \delta_{m0}(d/a)\mathbf{F}_{00}. \tag{A 1}$$

$\mathbf{F}_{00}$ drops out in the derivation of their (3.2) due to the constraint (2.8), and so their variational approximation to the far field is unaffected by their mistake. However, there is no way to determine $\mathbf{F}_{00}$, and hence the vertical force on the dock, from their variational formulation.

Moreover, it was suspected that Miles & Gilbert's technique for calculating the near field might lead to inaccurate values for the horizontal force and torque on the dock. From their variational formulation they can calculate the far field to $O(\epsilon^2)$ from a trial function which has the right shape to $O(\epsilon)$ (its magnitude cancels). With the trial function they use they probably get a very good approximation in this way to the amplitude of the far field; but, by using this to scale their trial function, they only calculate the wave forces to $O(\epsilon)$, and of course one doesn't know just how big an error $O(\epsilon)$ might represent.

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